A Simple Approach to Risk-Adjusted Performance

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Spring 2000

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Abstract

This note shows that the risk-adjusted performance measure developed by Modigliani and Modigliani [1997] can be derived by using a simple concept from geometry.

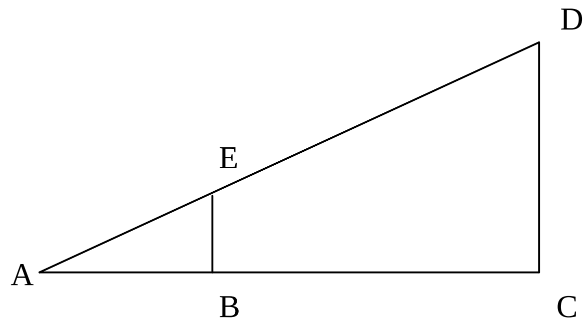
INTRODUCTION

Modigliani and Modigliani [1997] derived a risk-adjusted performance (RAP) measure by adjusting the risk of a particular portfolio so that it matches the risk of a market portfolio and then calculate the appropriate return for that portfolio. Unlike Sharpe measure [1966], Treynor measure [1965], and Jensen measure [1968], the unique feature of RAP is that it measures the performance of a portfolio in basis points, the traditional unit to measure return, and hence allow investors to compare the RAP of a portfolio directly with the return of a market portfolio. A high (low) RAP indicates that the portfolio has outperformed (underperformed) the market portfolio. Although Tam [1999] refers to the risk-adjusted performance by Modigliani and Modigliani [1997] as "M-squared," this note deliberately avoids the use of "M-squared" in order not to confuse readers with "M-squared" in Fong and Vasicek [1984] for a risk measurement of an immunized portfolio.

The objective of this note is to simplify the derivation of risk-adjusted performance measure (Equation (4) in Modigliani and Modigliani [1997]). The risk-adjusted performance measure can be derived by using a simple concept from geometry. This simplification will provide readers and investors with an additional insight to the risk-adjusted performance measure.

METHODOLOGY

This note uses the concept of similar triangles from geometry to show the derivation of RAP. From Nowlan and Washburn [1975, p. 266]:



Δ ABE ~ Δ ACD. Therefore, $\frac{AB}{AC} = \frac{BE}{CD}$

RESULTS

In order to avoid crowding out the graph, Figure 1 is a replica of Exhibit 1 in Modigliani and Modigliani [1997] but only for Portfolio 1, P_1, where the risk of this portfolio (σ_1) is greater than the risk of the market portfolio (σ_M). Although P_1 offers a higher return than the market portfolio, it also has a higher risk and thus r_1 is not compatible with r_M. RAP(1) is the risk-adjusted performance of P_1 after adjusting its risk to match the risk of market portfolio. The concept of similar triangles in geometry is applied to calculate RAP(1). Note that Figure 1 contains two triangles, both triangles are marked by heavy solid lines. Consequently,

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(Place Figure 1 about here)
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$$\frac{\sigma_M}{\sigma_1} = \frac{x}{r_1 - r_f} \tag{1}$$

$$x = \frac{\sigma_M(r_1 - r_f)}{\sigma_1} = \frac{\sigma_M}{\sigma_1}(r_1 - r_f)$$

$$RAP(1) = x + r_f \tag{2}$$

Equation (2) is the same as Equation (4) in Modigliani and Modigliani [1997].

As long as a portfolio has σ_1 that is greater than σ_M and r_1 that is higher than r_f, Equation (1) can be used to calculate the risk-adjusted performance of this portfolio. Applying the information in Table 1 to Equation (1), the RAP for Fidelity Magellan, for example, is:

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(Place Table 1 about here)
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$$\frac{7.2}{8.6} = \frac{x}{15.4 - 5.5} \rightarrow x = 8.29$$

Substituting x into Equation (2), the RAP for Fidelity Magellan is 8.29 + 5.5 = 13.79 ≈ 13.8, a result similar to the RAP for Fidelity Magellan in Modigliani and Modigliani [1997]. If Fidelity Magellan had the same level of risk as the market portfolio, it would had a 13.8% risk-adjusted return, a return that is lower than the return of the market portfolio (14.1%). For this reason, Fidelity Magellan had underperformed the market portfolio.

Figure 2 is also a replica of Exhibit 1 in Modigliani and Modigliani [1997] and contains the analysis for Portfolio 2, P_2, where the risk of this portfolio (σ_2) is less than the risk of the market portfolio (σ_M). Even tough P_2 offers a lower return than the market portfolio, it also has a lower risk and hence r_2 is not compatible with r_M. RAP(2) is the risk-adjusted performance of P_2 after adjusting its risk to match the risk of market portfolio. The same geometry approach is used to solve for RAP(2). Also note that Figure 2 contains two triangles, both triangles are marked by heavy solid lines. Consequently,

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(Place Figure 2 about here)
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$$\frac{\sigma_M - \sigma_2}{\sigma_M} = \frac{x - (r_2 - r_f)}{x} \tag{3}$$

$$x = \frac{\sigma_M (r_2 - r_f)}{\sigma_2} = \frac{\sigma_M}{\sigma_2}(r_2 - r_f)$$

$$RAP(2) = x + r_f \tag{4}$$

Equation (4) is also identical to Equation (4) in Modigliani and Modigliani [1997].

For a portfolio with σ_2 that is smaller than σ_M and r_2 that is higher than r_f, Equation (3) can be used to calculate the risk-adjusted performance of this portfolio. Applying the information in Table 1 to Equation (3), the RAP for Fidelity Puritan, for instance, is:

$$\frac{7.2 - 4.7}{7.2} = \frac{x - (12.0 - 5.5)}{x} \rightarrow x = 9.96$$

Substituting x into Equation (4), the RAP for Fidelity Puritan is 9.96 + 5.5 = 15.46 and is compared favorably to the RAP for Fidelity Puritan in Modigliani and Modigliani [1997]. If Fidelity Puritan had the same level of risk as the S&P 500, it would had a 15.5% risk-adjusted return, a return that is higher than the return of the S&P 500 (14.1%). Therefore, Fidelity Puritan had outperformed the S&P 500.

CONCLUSIONS

This note simplifies the derivation of risk-adjusted performance measure developed by Modigliani and Modigliani [1997]. The risk-adjusted performance measure can be derived by using a simple concept from geometry.

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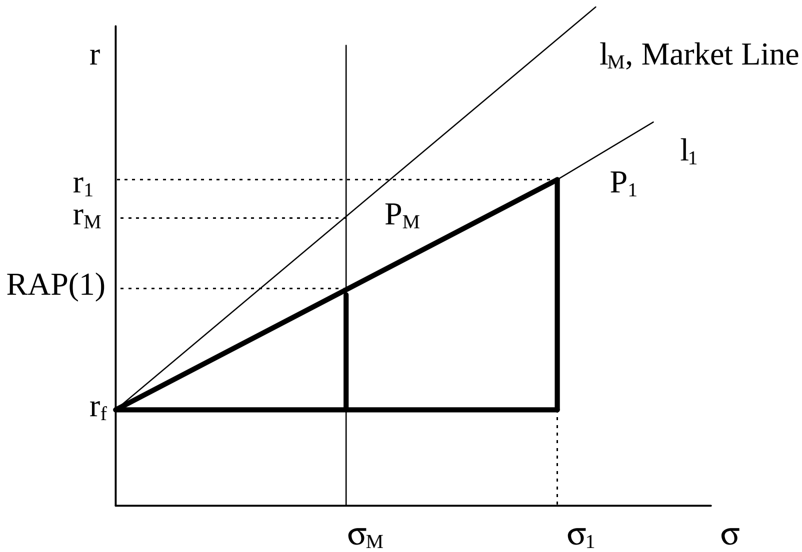
Figure 1. RAP for $\sigma_1 > \sigma_M$



where: r = return;

σ = standard deviation = risk;

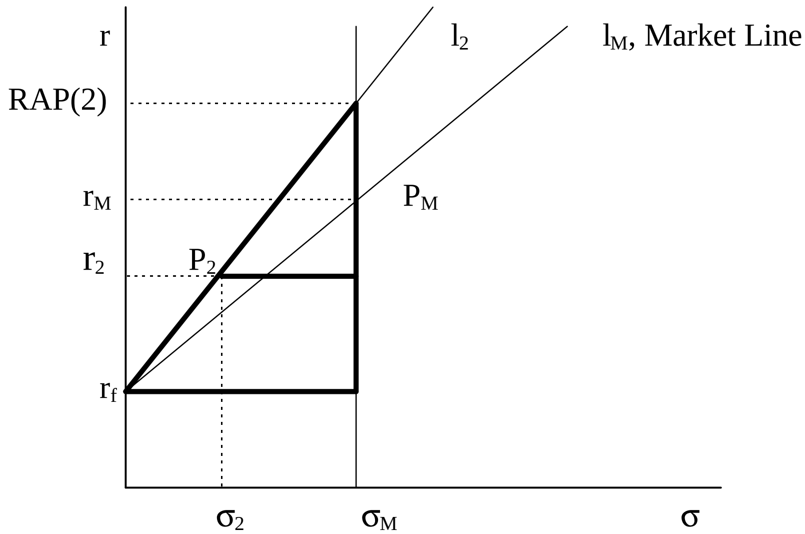
r_f = risk-free rate of return;

P_1 = Portfolio 1, with r_1 = return of Portfolio 1 and σ_1 = risk of Portfolio 1;

P_M = market portfolio, with r_M = return of market portfolio and σ_M = risk of market portfolio; and

RAP(1) = risk-adjusted performance of Portfolio 1.

Figure 2. RAP for $\sigma_2 < \sigma_M$



where: P_2 = Portfolio 2, with r_2 = return of Portfolio 2 and σ_2 = risk of Portfolio 2; and

RAP(2) = risk-adjusted performance of Portfolio 2.

Table 1. RAP: Analysis of Selected Mutual Funds

Mutual Funds (in order of total return)	Average Quarterly Total Return (at annual rate)	Quarterly Standard Deviation	Quarterly RAP (at annual rate)
S&P 500	14.1	7.2	14.1
AIM Constellation	19.7	12.3	13.9
20th Century Vista Investors	16.7	14.0	11.3
T. Rowe Price New Horizon	16.0	11.3	12.2
Fidelity Magellan	15.4	8.6	13.8
Vanguard Windsor	13.0	7.5	12.7
Fidelity Puritan	12.0	4.7	15.4
Income Fund of America	11.3	4.0	15.9
T-Bill	5.5		

Source: Modigliani and Modigliani [1997], p. 50.